MATH 1910, Fall 2016
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Sections: 201, 202

Name:__________________________

Chapter 6 and Exam Review

1. Calculate the following:

$$\frac{d}{dx}\int_1^{\sqrt{x}}\left(\frac{d}{dt}\int_1^{t^2}\frac{\sin s}{s}\,ds\right)\,dt.$$

2. Calculate:

$$\int (x+1)(x+3)^{2016}\,dx.$$

3. Evaluate the definite integral:

$$\int_1^e \frac{(\ln x)^2}{x}\,dx.$$

4. Find the flow rate through a tube of radius 4 cm, assuming that the velocity of fluid at a distance r centimeters from the center is $v(r) = (16 - r^2)$ cm/s.

5. a. Let M be the average value of $f(x) = x^4$ on $[0, 3]$. Find a value c in $[0, 3]$ such that $f(c) = M$.
   b. Let $f(x) = \sqrt{x}$. Find a value c in $[4, 9]$ such that $f(c)$ is equal to the average of f on $[4, 9]$.

6. Find the volume of the solid whose base is the unit circle $x^2 + y^2 = 1$, and the cross sections perpendicular to the x-axis are triangles whose height and base are equal.

7. Find the volume of the solid obtained by rotating the region enclosed by the graphs

$$y = x^2, \quad y = 12 - x, \quad x = 0,$$

about the line $y = -2$.

8. Use the shell method to compute the volume obtained by rotating the region enclosed by the graphs

$$y = 1 - |x - 1|, \quad y = 0$$

about the y-axis.

9. Calculate:

   a.

$$\int_0^1 xe^{-x^2/2}\,dx.$$

   b.

$$\int e^x \cos(e^x)\,dx.$$

10. Figure 2 shows a solid whose horizontal cross section at height y is a circle of radius $(1+y)^{-2}$ for $0 \le y \le H$. Find the volume of the solid as a function of H. What is the volume when $H = 1$?

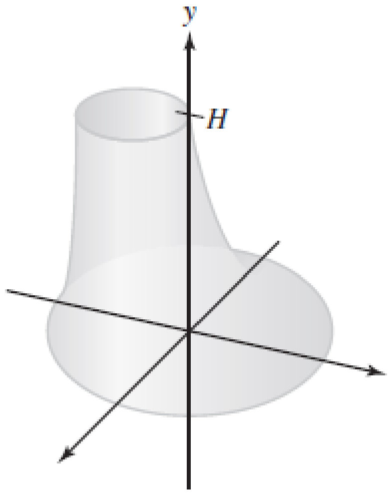

FIGURE 2

11. A tank of mass 20 kg containing 100 kg of water (density $1,000\ kg/m^3$) is raised vertically at a constant speed of 100 m/min for 1 min, during which time water is leaking at a rate of 40 kg/min. Calculate the total work performed in raising the container.

12. Water is pumped into a spherical tank of radius 2 m from a source located 1 m below a hole at the bottom (see Fig. 5). The density of water is $1,000\ kg/m^3$. Calculate the work $F(h)$ required to fill the tank to level h in meters in the sphere.

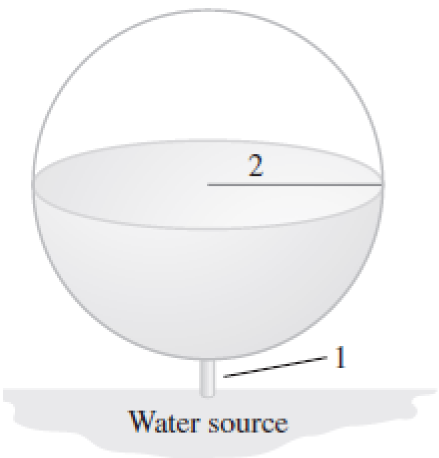

FIGURE 5